11.10

(a) FIND MACLAURIN SERIES FOR $f(x) = x^3 \cos 2x$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\cos 2x = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n}}{(2n)!}$$

$$x^3 \cos 2x = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n} x^3}{(2n)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n} x^{2n+3}}{(2n)!}$$

$$2n+3 = 2\underbrace{(n+1)}_{\text{TURN INTO } n}+1$$

$$(-1)^0 \frac{2^{2(0)} x^{2(0)+3}}{(2\cdot 0)!}$$

INDEX SHIFT: SHIFT n DOWN BY 1 INSIDE Σ, UP BY 1 ON LIMITS OF Σ

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^{2(n-1)} x^{2(n-1)+3}}{(2(n-1))!}$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^{2n-2} x^{2n+1}}{(2n-2)!}$$

$$(-1)^2 \frac{2^{2(1)-2} x^{2(1)+1}}{(2\cdot 1-2)!}$$

(b) FIND $f^{(9)}(0)$

$= -64 \cdot 9 \cdot 8 \cdot 7$

$$f(x) \approx T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$n=4$

IF $n=9$, $\frac{f^{(9)}(0)}{9!} \cancel{x^9} = \frac{(-1)^5 2^6 \cancel{x^9}}{6!}$ $\quad f^{(9)}(0) = \frac{-64}{\cancel{6!}} \cdot \cancel{9!}^{\,9\cdot 8\cdot 7}$

FIND THE MACLAURIN SERIES FOR $f(x) = (x^2+3x)e^{2x}$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$$

$$(x^2+3x)e^{2x} = \sum_{n=0}^{\infty} \frac{(x^2+3x)(2x)^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{2^n(x^{n+2}+3x^{n+1})}{n!}$$

$$= x^2e^{2x} + 3xe^{2x}$$

$$= \sum_{n=0}^{\infty} \frac{2^n x^{n+2}}{n!} + \sum_{n=0}^{\infty} \frac{3\cdot 2^n x^{n+1}}{n!}$$

INDEX SHIFT TO GET x^n

INSIDE DOWN 2
LIMITS UP 2

INSIDE DOWN 1
LIMITS UP 1

$$= \sum_{n=2}^{\infty} \frac{2^{n-2}x^n}{(n-2)!} + \sum_{n=1}^{\infty} \frac{3\cdot 2^{n-1}x^n}{(n-1)!}$$

$$= \sum_{n=2}^{\infty} \frac{2^{n-2}x^n}{(n-2)!} + \sum_{n=2}^{\infty} \frac{3\cdot 2^{n-1}x^n}{(n-1)!} + \frac{3\cdot 2^0 x^1}{0!}$$

$$= 3x + \sum_{n=2}^{\infty} \left(\frac{2^{n-2}}{(n-2)!} + \frac{3\cdot 2^{n-1}}{(n-1)!}\right) x^n$$

CONT'D

$$\frac{2^{n-2}}{(n-2)!} + \frac{3 \cdot 2^{n-1}}{(n-1)!}$$

$$= \frac{2^{n-2}}{(n-1)!}\left(1(n-1) + 3 \cdot 2 \right)$$

$$= \frac{(n+5)\,2^{n-2}}{(n-1)!}$$

$$\frac{1}{(n-2)!} = \frac{1}{(n-2)!}\,\frac{(n-1)}{(n-1)}$$

$$= \frac{n-1}{(n-1)!}$$

$$\text{so } f(x) \approx T(x) = 3x + \sum_{n=2}^{\infty} \frac{(n+5)\,2^{n-2}}{(n-1)!}x^n = \sum_{n=1}^{\infty} \frac{(n+5)\,2^{n-2}}{(n-1)!}x^n$$

IF $n=1$ IN $\frac{6 \cdot 2^{-1}}{0!}x^1 = 3x$

APPROXIMATE $\int_1^3 \frac{\sin x}{x}\,dx$ USING FIRST 3 TERMS OF ~~I.S.~~ M.S. FOR $\sin x$

NON-ZERO

$n=0,1,2$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\frac{\sin x}{x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!}$$

$$\int_1^3 \frac{\sin x}{x}\,dx = \sum_{n=0}^{\infty} \int \frac{(-1)^n x^{2n}}{(2n+1)!}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}\Bigg|_{x=1}^{x=3}$$

$$\approx \frac{(-1)^0}{1\cdot 1!}(3^1 - 1^1) + \frac{(-1)^1}{3\cdot 3!}(3^3 - 1^3) + \frac{(-1)^2}{5\cdot 5!}(3^5 - 1^5)$$

$$= 2 - \frac{\cancel{26}\ 13}{\cancel{18}\ 9} + \frac{\cancel{242}\ 121}{5\cdot 5\cdot 4\cdot 3\cdot \cancel{2}\ 1}$$

$$= 2 - \frac{13}{9} + \frac{121}{300}$$

$$= \frac{1800 - 1300 + 363}{900} = \frac{863}{900}$$